2013 13th International Conference on Control, Automation and Systems (ICCAS 2013)
Oct. 20-23, 2013 in Kimdaejung Convention Center, Gwangju, Korea

Graph and Controller Design for Disturbance Attenuation in Consensus Networks

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Abstract: Disturbance attenuation in consensus networks has been recently studied in the literature. In this paper, we provide illustrative examples of graph and controller design for disturbance attenuation in undirected consensus networks of identical linear time-invariant systems subject to exogenous disturbance. We first summarize some existing results on disturbance attenuation in such consensus networks, and then solve specific graph and controller design examples based on the results.

Keywords: Consensus; undirected graph; disturbance attenuation; linear matrix inequality.

1. INTRODUCTION

For a consensus network, if individual systems in the network are subject to exogenous disturbance, they may not reach perfect consensus. In such a case, it is important to consider how to design the network in order to attenuate the disturbance and thereby achieve a prescribed performance relative to perfect consensus.

The $\mathcal{H}_\infty$ norm of the transfer function from the disturbance vector to the disagreement vector of a consensus network has been usually taken for disturbance attenuation performance metric. Liu *et al.* have studied local controller design for an undirected network of identical linear time-invariant (LTI) systems subject to exogenous disturbance [1, 2]. For a prescribed performance, they have formulated an $\mathcal{H}_\infty$ suboptimal problem, and provided a linear matrix inequality (LMI) to find local controllers that ensure the performance. A similar result for an undirected network of identical second-order LTI systems is found in [3]. Li *et al.* have also addressed a similar problem assuming that some individual systems can measure their own state [4].

Taking into account that the performance depends not only on the local controller but also on the network graph, Oh *et al.* have further studied both graph weight specification and local controller design to enhance the performance [5]. First, they have formulated an $\mathcal{H}_\infty$ suboptimal control problem for an undirected network of identical LTI systems to ensure a prescribed performance assuming that network graph topology is given but edge weights are variables belonging to a convex set. By an LMI, they have characterized a condition under which the performance is enhanced by maximizing the second smallest eigenvalue of the network graph Laplacian. Second, for a consensus network with existing interconnection that has fixed weights, they have formulated two $\mathcal{H}_\infty$ suboptimal problems based on distributed and decentralized controllers, respectively, and provided algorithms for the design of these controllers to ensure a prescribed performance. Both types of the controllers can be designed by solving LMI feasibility problems.

In this paper, we solve several specific examples of graph and controller design for enhancing disturbance attenuation performance in undirected consensus networks of identical LTI systems subject to exogenous disturbance. We first summarize some existing results on disturbance attenuation in undirected consensus networks. Based on the results, particularly result in [5], we then illustratively solve some specific design examples. For a network of five linear coupled oscillators, we specify edge weights of its network graph, thereby maximizing disturbance attenuation performance. For a network of five linear coupled oscillators that has existing interconnection, we design distributed and decentralized controllers that ensure the disturbance attenuation performance.

The rest of this paper is organized as follows. In Section 2, we review the algebraic graph theory concepts. In Section 3, we summarize the existing results on disturbance attenuation in undirected consensus networks. Graph and local controller design examples are illustratively solved in Section 4. Section 5 concludes the paper.

2. PRELIMINARIES

The set of real numbers is denoted by $\mathbb{R}$. The set of nonnegative (respectively, positive) real numbers is denoted by $\bar{\mathbb{R}}_+$ (respectively, $\mathbb{R}_+$). We denote $[1 \cdots 1]^T \in \mathbb{R}^n$ by 1_n. The $n \times n$ identity matrix is denoted by I_n. For $A \in \mathbb{R}^{n \times n}$, we denote the positive definiteness (respectively, positive semidefiniteness) of A by $A \succ 0$ (respectively, $A \succeq 0$). Further, by $A \prec 0$ (respectively, $A \preceq 0$), we denote the negative definiteness (respectively, negative semi-definiteness) of A. For any matrix A, A^T denotes the transpose of A. For any two matrices, $A \otimes B$ denotes the Kronecker product of A and B.

A weighted undirected graph (or simply graph) $\mathcal{G}$ is defined as a triple $\mathcal{G} := (\mathcal{V}, \mathcal{E}, \mathcal{W})$, where $\mathcal{V}$ denotes the set of nodes, $\mathcal{E} \subset \{\{i,j\} : i,j \in \mathcal{V}\}$ denotes the set of edges, and $\mathcal{W} : \mathcal{E} \to \bar{\mathbb{R}}_+$ denotes a map assigning nonnegative real weights to the edges. Assume there are no self-loops, i.e., for any $i \in \mathcal{V}$, $\{i,i\} \notin \mathcal{E}$. The nonnegative value $\mathcal{W}(\{i,j\})$ assigned to $\{i,j\} \in \mathcal{E}$ is called the weight of the edge. If $\{i,j\} \in \mathcal{E}$, j (respectively, i) is said to be a neighbor of i (respectively, j). The set of neighbors of $i \in \mathcal{V}$ is defined as $\mathcal{N}_i := \{j \in \mathcal{V} : \{i,j\} \in \mathcal{E}\}$. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N \times N}$ of $\mathcal{G}$ is defined as

$$l_{ij} := \begin{cases} \sum_{j \in \mathcal{N}_i} w_{ij}, & i = j, \\ -w_{ij}, & \{i,j\} \in \mathcal{E}, \\ 0, & \{i,j\} \notin \mathcal{E}, \end{cases}$$

where $w_{ij} := \mathcal{W}(\{i,j\})$ for any $\{i,j\} \in \mathcal{E}$.

Consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ with M edges. By an ordering, we represent the edge set by $\mathcal{E} = \{e_1, \ldots, e_M\}$, where e_k is a node pair $\{i,j\} \in \mathcal{E}$ for all $k = 1, \ldots, M$. Then we define

$$w := [\mathcal{W}(e_1) \cdots \mathcal{W}(e_M)]^T \in \bar{\mathbb{R}}_+^{\mathbb{M}}. \tag{1}$$

We refer to w as the edge weight of $\mathcal{G}$.

3. SUMMARY OF PREVIOUS RESULTS

3.1 Graph design for disturbance attenuation

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ be an undirected graph with N nodes and M edges. Consider the following consensus network over $\mathcal{G}$:

$$\dot{x}_i = Ax_i + F \sum_{j \in \mathcal{N}_i} w_{ij}(x_j - x_i) + Ed_i, \tag{2a}$$

$$z_i = x_i - \frac{1}{N} \sum_{j=1}^{N} x_j, \tag{2b}$$

where $x_i \in \mathbb{R}^n$, $z_i \in \mathbb{R}^q$, and $d_i \in \mathbb{R}^l$ denote the state, the output, and the exogenous disturbance, respectively, of node i, and $A \in \mathbb{R}^{n \times n}$, $E \in \mathbb{R}^{n \times l}$, and $F \in \mathbb{R}^{n \times p}$ are constant matrices. We assume that $(\mathcal{V}, \mathcal{E})$ is fixed while $\mathcal{W}$ can be designed. Thus the edge weight w of $\mathcal{G}$ defined in (1) is the design variable. The overall equation for the network (2) is written as

$$\dot{x} = (I_N \otimes A - L \otimes F)x + (I_N \otimes E)d, \tag{3a}$$

$$z = (\bar{L} \otimes I_n)x, \tag{3b}$$

where $x := [x_1^T \cdots x_N^T]^T$, $z := [z_1^T \cdots, z_N^T]^T$, $d := [d_1^T \cdots d_N^T]^T$, L is the Laplacian of $\mathcal{G}$, and $\bar{L} := I_N - 1_N 1_N^T / N$. Let us call z the disagreement vector of x.

It is said that the network (3) asymptotically reaches consensus when $x_i(t) - x_j(t) \to 0$ as $t \to \infty$ for all $i,j \in \mathcal{V}$. The network (3) may not reach consensus in the presence of the exogenous disturbance d. Thus it is important to design w appropriately so that the network (3) becomes less sensitive to d. Let $T_{zd}(s)$ be the transfer function matrix from d to z. Then we take the $\mathcal{H}_\infty$ norm of $T_{zd}(s)$ for the disturbance attenuation performance metric of the network (3) [1–5].

Let λ_2 be the second smallest eigenvalue of L. The following theorem characterizes a network condition under which a prescribed performance is ensured [5]:

Theorem 1: [5] Let $\gamma > 0$ be given. The network (3) asymptotically reaches consensus when $d \equiv 0$ and $\|T_{zd}(s)\|_\infty \leq \gamma$ if there exists $P = P^T \succ 0$ such that

$$\begin{bmatrix} \Omega_2 & PE \\ E^T P & -\gamma^2 I_n \end{bmatrix} \preceq 0, \tag{4a}$$

$$F^T P + PF \succeq 0, \tag{4b}$$

where $\Omega_2 = (A - \lambda_2 F)^T P + P(A - \lambda_2 F) + I_n$.

From (4), λ_2 obviously needs to be increased for enhancing the disturbance attenuation performance. Thus, if the network (3) satisfies the LMI (4), the graph weight design is reduced into maximizing λ_2.

Meanwhile, the norm of L is proportional to the magnitude of interaction term, $-(L \otimes F)x$, and thus it is not desirable to increase it arbitrarily. Furthermore, since the eigenvalues of L are positive homogeneous functions of w, it is required to impose the following constraints on w to make graph weight design sensible [7]:

$$c^T w = \sum_{k=1}^{M} c_k w_k \leq b_c, \tag{5}$$

where $c \in \mathbb{R}_+^M$. Here, b_c can be understood as the total budget for the graph edge while c_k are the marginal costs.

Since λ_2 is a symmetric concave function of w, maximization of λ_2 is posed as the following convex optimization problem [6]:

$$\begin{array}{l} \text{maximize } \lambda_2(w) \\ \text{subject to } c^T w \leq b_c. \end{array} \tag{6}$$

The optimization problem (6) can be transformed into a semidefinite programming problem, which can be efficiently solved [7].

3.2 Controller design for disturbance attenuation

Consider the following network over an undirected graph $\mathcal{G}_p = (\mathcal{V}, \mathcal{E}_p, \mathcal{W}_p)$ with N nodes:

$$\dot{x}_i = Ax_i + Bu_i + F \sum_{j \in N_{p,i}} w_{p,ij}(x_j - x_i) + Ed_i, \tag{7a}$$

$$z_i = x_i - \frac{1}{N} \sum_{j=1}^{N} x_j, \tag{7b}$$

where $x_i \in \mathbb{R}^n$, $z_i \in \mathbb{R}^q$, $u_i \in \mathbb{R}^m$, and $d_i \in \mathbb{R}^l$ denote the state, the measurement, the output, the control input, and the exogenous disturbance, respectively, of node i, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $E \in \mathbb{R}^{n\times l}$, and $F \in \mathbb{R}^{n\times p}$ are constant matrices. Assume $\mathcal{G}_p$ is given a priori. The overall equation for the network (7) is written as

$$\begin{aligned} \dot{x} &= (I_N \otimes A - L_p \otimes F)x + (I_N \otimes B)u \\ &\quad + (I_N \otimes E)d, && (8a) \\ z &= (\bar{L} \otimes I_n)x, && (8b) \end{aligned}$$

where L_p is the Laplacian matrix of $\mathcal{G}_p$.

For the network (8), we consider two types of controllers. The first controller is the distributed controller. Let $\mathcal{G}_c = (\mathcal{V}, \mathcal{E}_c, \mathcal{W}_c)$. The distributed controller has the form $u_i = K \sum_{j\in N_{c,i}} w_{c,ij}(x_j - x_i)$, where $K \in \mathbb{R}^{n\times n}$. Thus u can be written as

$$u = -(L_c \otimes K)x, \tag{9}$$

where L_c is the Laplacian matrix of $\mathcal{G}_c$. It is assumed that the controller for node i has a neighborhood $\mathcal{N}_{c,i}$ which is possibly different than $\mathcal{N}_{p,i}$.

The second controller is the decentralized controller of the form $u_i = -Kx_i$ for all $i = 1, \ldots, N$, where $K \in \mathbb{R}^{n\times n}$ is the control gain, which is written in a vectorial form

$$u = -(I_N \otimes K)x. \tag{10}$$

For the decentralized controller (10), $\|K\|_2$ is the maximum gain for the measurement x. Since such a measurement contains noises in reality, it is not desirable to increase $\|K\|_2$ arbitrarily because high gains usually cause high sensitivity to noises. Further high gains might give rise to input saturation. In this regard, it is natural to impose a limitation constraint on $\|K\|_2$ as

$$\|K\|_2 \le b_K. \tag{11}$$

Let $w_c = [w_{c,1} \cdots w_{c,M_c}]^T \in \bar{\mathbb{R}}_+^{M_c}$ be the edge weight of $\mathcal{G}_c$, where M_c is the cardinality of $\mathcal{E}_c$. Based on the same reason, for the distributed controller (9), we impose the following constraint:

$$w_{c,k}\|K\|_2 \le b_{Kw},\ k = 1, \ldots, M_c. \tag{12}$$

3.2.1 Design of distributed control networks

If each individual system (7) has a distributed controller (9), the overall equation is arranged as

$$\begin{aligned} \dot{x} &= (I_N \otimes A - L_p \otimes F - L_c \otimes BK)x \\ &\quad + (I_N \otimes E)d, && (13a) \\ z &= (\bar{L} \otimes I_n)x. && (13b) \end{aligned}$$

The following theorem presents an LMI to find the distributed controller K without considering the constraint (12) [5]:

Theorem 2: [5] Let $\gamma > 0$ be given. The network (13) asymptotically reaches consensus when $d \equiv 0$ and $\|T_{zd}(s)\|_\infty \le \gamma$ if there exist Y and $Q = Q^T \succ 0$ such that

$$\begin{bmatrix} \Omega_{2,2} & Q \\ Q & -\gamma^2 I_n \end{bmatrix} \preceq 0, \tag{14a}$$

$$BY + Y^T B^T \succeq 0, \tag{14b}$$

$$FQ + QF^T \succeq 0, \tag{14c}$$

where $\Omega_{2,2} = Q(A - \lambda_{p,2}F)^T + (A - \lambda_{p,2}F)Q - \lambda_{c,2}(BY + Y^T B^T) + EE^T$.

Note that, based on **Theorem 2**, K can be designed as $K = YQ^{-1}$.

3.2.2 Design of decentralized control networks

If each individual system (7) has a decentralized controller (10), the overall equation is arranged as

$$\begin{aligned} \dot{x} &= (I_N \otimes A - L_p \otimes F - I_N \otimes BK)\,x \\ &\quad + (I_N \otimes E)d, && (15a) \\ z &= \bar{L}x. && (15b) \end{aligned}$$

The following theorem presents an LMI to find the decentralized controller K without considering the constraint (11) [5]:

Theorem 3: [5] Let $\gamma > 0$ be given. The network (15) asymptotically reaches consensus when $d \equiv 0$, and $\|T_{zd}(s)\|_\infty \le \gamma$ if there exist Y and $Q = Q^T \succ 0$ such that

$$\begin{bmatrix} \Omega_2 & Q \\ Q & -\gamma^2 I_n \end{bmatrix} \preceq 0, \tag{16a}$$

$$FQ + QF^T \succeq 0, \tag{16b}$$

where $\Omega_2 = (A - \lambda_{p,2}F - BK)^T P + P(A - \lambda_{p,2}F - BK) - (BY + Y^T B^T) + EE^T$.

Note that, based on **Theorem 3**, K can be designed as $K = YQ^{-1}$.

4. GRAPH AND CONTROLLER DESIGN EXAMPLES

We provide several design examples in this section. To solve LMI problems, we use `YALMIP` [9]. To maximize the second smallest eigenvalue of a graph, we use `CVX`, which is a package for specifying and solving convex programs [10].

4.1 Graph design example

Consider the network of N linear coupled oscillators over $\mathcal{G}$:

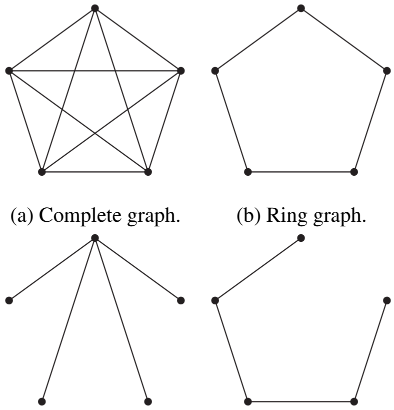
$$\dot{x}_i = Ax_i + F \sum_{j\in N_i} w_{ij}(x_j - x_i) + Ed_i,\ i = 1, \ldots, N,$$

where $x_i \in \mathbb{R}^2$ and A, E, and F are defined as follows:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},\ E = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Table 1: Result for the graph design example.

	Complete graph	Ring graph	Star graph	Tree graph
$w^\star$	$5 \times 1_{10}$	$10 \times 1_5$	$12.5 \times 1_4$	$[10\ 15\ 15\ 10]^T$
$\lambda_2^\star$	25.0	13.82	12.5	5.0
γ_{min}	0.0565	0.1021	0.1128	0.2774
$\|T_{zd}(s)\|_\infty$	0.0565	0.1021	0.1128	0.2774



(a) Complete graph. (b) Ring graph.

(c) Star graph. (d) Tree graph.

Fig. 1: Four types of graphs.

For these linear coupled oscillators, if $\gamma > 0$ is given, we can find $\lambda_{2,min}$ that satisfies the LMI condition (4). For instance, if $\gamma = 0.05$, then the LMI condition (4) is feasible when $\lambda_{2,min} \geq 23.9$. That is, if we can increase the second smallest eigenvalue of $\mathcal{G}$ greater than or equal to 23.9, then it is ensured that $T_{zd}(s) \leq 0.05$.

Let $N = 5$ and $b_c = 50$. Given different graphs in Fig. 1, we estimate the distrubance attenuation performance of the network. First, we find the maximum second smallest eigenvalue $\lambda_2^\star$ for each case by solving the convex optimization problem (6). For simplicity, we assume that all the entries of c are one for each case. Second, after plugging $\lambda_2^\star$ into (4), we then solve the following optimization problem:

$$\begin{aligned} &\text{maximize } \gamma \\ &\text{subject to } P \in \mathbb{R}^n, P = P^T \succ 0, (4). \end{aligned} \tag{17}$$

Table 1 shows the result. In Table 1, γ_{min} is the solution of the optimization problem (17) while $\|T_{zd}\|_\infty$ is the actual $\mathcal{H}_\infty$ norm of the network for each graph case. From Table 1, we can see that greater $\lambda_2^\star$ yields less γ_{min}, which implies that more interconnections lead to better disturbance attenuation performance. This is because the LMI condition (4) characterizes the condition under which interconnections are beneficial to consensus. Further, according to **Theorem 1**, it is desirable for a network to have greater $\lambda_2^\star$ under the network condition. In this sense, the best disturbance attenuation performance estimate is ensured when $(\mathcal{V}, \mathcal{E})$ is a complete graph.

4.2 Controller design example

We next consider the design of distributed and decentralized controllers for the network of five linear coupled oscillators over $\mathcal{G}_p$:

$$\begin{aligned} \dot{x}_i = Ax_i + F \sum_{j \in \mathcal{N}_{p,i}} w_{p,ij}(x_j - x_i) + Bu_i \\ + Ed_i,\ i = 1, \ldots, 5. \end{aligned}$$

where $x_i \in \mathbb{R}^2$ and A, B, E, and F are defined as follows:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$
$$E = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

We assume that $\mathcal{G}_p$ is a ring graph with $w_p = 1_5$. Thus we have $\lambda_{p,2} = 1.382$.

To check the constraint (12), we consider the condition $\|K\|_2 = \|YQ^{-1}\|_2 \leq \alpha$ with $\alpha = b_{Kw}/k_{w_c}$ in addition to (14) assuming that $w_c = k_{w_c} 1_{M_c}$. Such a norm boundedness condition is satisfied if the following LMI holds [5]:

$$\begin{bmatrix} 2Q - I_n & Y^T \\ Y & \alpha^2 I_n \end{bmatrix} \succeq 0, \tag{18}$$

which is equivalent to $Q \succeq (1/2)I_n$ and $\alpha^2(2Q - I_n) - Y^T Y \succeq 0$. From that $Q^2 \succeq 2Q - I_n$, we have $\alpha^2 Q^2 - Y^T Y \succeq 0$. It then follows from the congruence transformation [8] that $\alpha^2 I_n - Q^{-1} Y^T Y Q^{-1} \succeq 0$, which leads to $\|YQ^{-1}\|_2 \leq \alpha$. In short, to check the constraint (12), we consider (18) with $\alpha = b_{Kw}/k_{w_c}$ in addition to (14).

For the design of a distributed controller, we assume that $b_{Kw} = 40$ and information interconnection topologies are given as depicted in Fig. 1. In order to estimate the disturbance attenuation performance for each information interconnection topology based on **Theorem 2**, we find $\lambda_{c,2}$ by setting $w_c = k_{w_c} 1_{M_c}$ with different values of k_{w_c}, plug $\lambda_{c,2}$ into (14), and then solve the following optimization problem with $\alpha = k_{w_c} b_{Kw}$:

$$\begin{aligned} &\text{maximize } \gamma \\ &\text{subject to } Y \in \mathbb{R}^n, Q \in \mathbb{R}^n, Q = Q^T \succ 0, \\ &\qquad (14), (18). \end{aligned} \tag{19}$$

Table 2 shows the result. In Table 2, we assume $w_c = 1_{M_c}$ for each case because the solution of (19) does not

Table 2: Result for the distributed controller design example.

	Complete graph	Ring graph	Star graph	Tree graph
w_c	1_{10}	1_5	1_4	1_4
$\lambda_{c,2}$	5	1.382	1	0.382
K	$\begin{bmatrix} 0.1908 & 34.5966 \\ 34.7841 & -0.1740 \end{bmatrix}$	$\begin{bmatrix} 0.6425 & 34.4697 \\ 35.0695 & -0.5626 \end{bmatrix}$	$\begin{bmatrix} 0.9244 & 34.4434 \\ 35.3193 & -0.8597 \end{bmatrix}$	$\begin{bmatrix} 2.7760 & 33.8264 \\ 36.2309 & -2.5066 \end{bmatrix}$
γ_{min}	0.0439	0.0835	0.0982	0.1592
$\|T_{zd}(s)\|_\infty$	0.0095	0.0286	0.0440	0.0862

depend on the value of k_{w_c} in this example, though it might does in general.

The results obtained based on **Theorem 2** are conservative. When $L_c = kL_p$ for some $k > 0$, we can easily find the actual norm $\|T_{zd}(s)\|_\infty$ by decomposing the network of the linear coupled oscillators into low-order subsystems [1,4,5]. When $w_c = 1_5$, we have

$$K = \begin{bmatrix} 0.4558 & 34.5675 \\ 35.0033 & -0.3995 \end{bmatrix},$$

and $\|T_{zd}(s)\|_\infty = 0.0286$.

To design a decentralized controller based on **Theorem 3**, we assume that $b_K = 40$. To estimate the disturbance attenuation performance, we solve the following optimization problem with $\alpha = b_K$:

$$\begin{aligned} &\text{maximize } \gamma \\ &\text{subject to } Y \in \mathbb{R}^n, Q \in \mathbb{R}^n, Q = Q^T \succ 0, \\ &\qquad (16), (18). \end{aligned} \tag{20}$$

As a solution of (20), we have $\gamma_{min} = 0.0982$ with

$$K = \begin{bmatrix} 1.2870 & 34.1670 \\ 35.2262 & -0.9006 \end{bmatrix}.$$

Further, the actual $\mathcal{H}_\infty$ norm is given by $\|T_{zd}(s)\| = 0.0389$.

Two comments are in order on the above results. First, stronger information interconnections tend to ensure better disturbance attenuation performance estimate. This is because the LMI (14) characterizes a condition under which interconnections are beneficial to consensus. Second, decentralized controllers are not always better than distributed controllers in terms of disturbance attenuation under our problem setup. In the above example, the distributed controller provides better performance estimate when the information interconnection is a complete or ring graph.

5. CONCLUSION

We studied disturbance attenuation in undirected consensus networks of identical LTI systems subject to exogeous disturbance. We provided a summary of the existing results, and then illustratively solved some graph and controller design examples.

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